MEASUREMENTS OF CONVECTIVE HEAT TRANSFER FROM A HORIZONTAL CYLINDER ROTATING IN A TANK OF WATER

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Abstract-The present paper deals with measurements of heat transfer from a horizontal cylinder rotating in water. The experimental results have been correlated by the equation $Nu = 0.133Re^{2/3}$. $Pr^{1/3}$ for a range of rotating Reynolds numbers from 1000 to 46 000, and Prandtl numbers from 2.2 to 6.4. This equation compares very well with the experimental and theoretical information available for air, water and oil in published works.

The analogy suggested by Anderson and Saunders between natural convection from a horizontal plate and the present type of flow has been used to predict the Nusselt numbers. The analogy solution and the present experimental results have been found to compare very well with each other.

NOMENCLATURE

- specific heat, KJ/kg degC; c_p
- diameter of test section, mm; d_{\cdot}
- inner diameter of test section, m; d_i
- $E.$ voltage, V;
- friction coefficient, dimensionless; f,
- acceleration due to gravity, m/s^2 ; g,
- current, A ; I.
- bulk temperature, °C; t_{R_1}
- inner wall temperature, "C; $_{tm}$.
- outer wall temperature, "C; t_{wo}
- Grashof number, dimensionless; Gr.
- $Nu.$ Nusselt number, dimensionless;
- Prandtl number, dimensionless; $Pr.$
- Re. Reynolds number, dimensionless; or
- heat-transfer coefficient, KJ/m² s degC; α .
- β, coefficient of thermal expansion, $degC^{-1}$:
- λ .
- ω .
- ρ ,
- μ,
- θ .

CONVECTIVE heat transfer from a horizontal cylinder rotating in air has earlier been studied by several investigators. Information about other fluids, however, is very scarce, and the purpose of the present paper is to present data with a cylinder rotating in a tank of water, and to show the effects of Prandtl number on this type of heat transfer. The heat-transfer coefficient for the cylinder is assumed to be a function of the following independent variables

$$
a = f(d, g, c_p, \omega, \beta, \rho, \mu, \lambda, \theta). \tag{1}
$$

Using dimensional analysis equation (1) can be reduced to

$$
\frac{\alpha d}{\lambda} = f_1 \left(\frac{\omega d^2 \rho}{2\mu}, \frac{g d^3 \beta \theta \rho^2}{\mu^2}, \frac{\mu c_p}{\lambda} \right) \tag{2}
$$

$Nu = f₂(Re, Gr, Pr).$ (3)

thermal conductivity, KJ/m s degC;
angular velocity, s^{-1} ;
angular velocity, s^{-1} ; angular velocity, s^{-1} ;
density, $\frac{1}{2}$;
density, $\frac{1}{2}$;
density, $\frac{1}{2}$; published works are those of Anderson and viscosity, kg/m s; Saunders [1], Etemad [2], Dropkin and Carmi temperature difference, degC. $[3]$ and Kays and Bjorklund $[4]$ for air, and that of Seban and Johnson [5] for oil and water. The present section briefly describes these 1. INTRODUCTION studies and other information available which is of importance to the subject.

> Anderson and Saunders [l] investigated the heat transfer from horizontal cylinders, 1.0 , 1.8

and 3.9 inches in diameter, each 2 ft long rotating in still air, and found that up to a critical value of the Reynolds number, based on surface velocity, the Nusselt number is almost independent of the Reynolds number, and the rate of heat transfer is then mainly determined by the free convection. Using theoretical considerations the critical Reynolds number was found to be equal to

$$
Re_{Cr} = 1.09 \text{ } Gr^{1/2}. \tag{4}
$$

Above the critical Reynolds number it was found that the Nusselt number increased with the Reynolds number and that the Grashof number had a negligible effect on the rate of heat transfer.

Anderson and Saunders suggested that the flow set up by the rotating cylinder above the critical Reynolds number is analogous in many respects to the irregular flow which occurs in free convection above a heated horizontal plate facing upwards. Using this analogy they derived the following expression for heat transfer from a cylinder rotating in still air.

$$
Nu = 0.10 \; Re^{2/3}.\tag{5}
$$

This equation compared excellently with the measurements.

Etemad [2] studied experimentally the heat transfer and flow around horizontal cylinders, 2- and $2\frac{1}{2}$ inches in diameter, rotating in air. A range of Reynolds numbers from 0 to 65 400 was studied. From interferometric observations he found that the laminar Couette motion broke down at a critical Reynolds number of 900 compared with 1080 computed from the relation established by Anderson and Saunders. The interferometric pictures also showed that the secondary flow above the critical Reynolds number bore some resemblance to the secondary flow between two concentric cylinders, when the inner cylinder was rotated. The latter type of flow has been studied by Taylor [6], Kaye and Elgar [7] and others. Etemad found further that up to a Reynolds number of 14 500 the secondary flow remained in steady motion. Above this value the secondary flow broke down and the flow became turbulent. The heat-transfer results by Etemad compared excellently with the data

of Anderson and Saunders. For Reynolds nun) bers above 8000, the heat-transfer rates were independent of the Grashof number and the following equation correlated the experimental data.

$$
Nu = 0.076 \, Re^{0.70}.\tag{6}
$$

For Reynolds numbers below 1000 the Nusselt numbers depended almost entirely on the Grashof numbers, and in the intermediate range. between 1000 and 8000 both the Grashofand the Reynolds numbers influenced the rate of heat transfer and the following correlation was recommended

$$
Nu = 0.11 [(0.5 \, Re^2 + Gr) \cdot Pr]^{0.35}. \tag{7}
$$

Dropkin and Carmi [3] measured the heattransfer rates from horizontal rotating cylinder< to ambient air for Reynolds numbers up to 433 000. The diameters employed were 3.25 and 4.50 in. For Reynolds numbers larger than $15\,000$ they recommended the following equation

$$
Nu = 0.073 \, Re^{0.7} \tag{8}
$$

which compares extremely well with the results mentioned earlier. In the region where both rotation and natural convection influenced the heat transfer their data were correlated by the equation

$$
Nu = 0.095 (0.5 \, Re^2 - Gr)^{0.35}.
$$
 (9)

Kays and Bjorklund [4J measured the heat transfer from a horizontal cylinder rotating in air with and without crossfiou. In the case of zero crossflow their results compared very well with the investigations previously mentioned. This case was also investigated theoretically by means of the momentum and heat-transfer analogy, and it was found that the Nusselt number could be predicted by the equation

$$
Nu = \frac{Re \cdot Pr \cdot \sqrt{(f/2)}}{5 Pr + 5 \ln(3 Pr + 1) + [1/\sqrt{(f/2)}]} - 12
$$
\n(10)

For estimating the friction coefficient, f , the use of the data by Theodersen and Regier $[8]$ was recommended. In the case of air where $Pr =$ 0.72 , the analogy solution agreed very well with the experimental results.

Seban and Johnson [5] studied experimentally the heat transfer from a horizontal cylinder, $2\frac{1}{2}$ inches in diameter rotating in a tank of oil. The results embraced a Prandtl number range from 130 to 670 and Reynolds numbers up to 15 000, and showed an increasing dependence of free convection heat transfer on rotation as the Prandtl was increased by reducing the oil temperature. At higher rotative speeds, where the flow became turbulent and the free convection effects vanished, the results were correlated by plotting $Nu/Pr^{0.356}$ versus the Reynolds number. Some additional measurements were also performed with water in the tank.

3. DESCRIPTION OF APPARATUS

The details of the rotor is shown in Fig. 1, and a schematic view of the apparatus is reproduced in Fig. 2. The electric resistance heated test section consisted of a polished stainless steel tube, 300 mm in length, 10.05 mm in outer diameter and with wall thickness of O-5 mm. At both ends the tube was silver soldered to copper rods which penetrated to the exterior through seals mounted in the walls of the stainless steel water container. In order to avoid electrolytic exchange of copper ions, the copper rods were covered by stainless steel tubes. On the outside of the water container, the copper rods were bolted to heavy copper cylinders 60 mm in diameter. Sixteen carbon brushes with a 20×20 mm cross section rested against each of the copper cylinders. This arrangement permitted 3000 A or approximately 100 kW to be supplied to the test section. The power came from a direct current generator which delivered

FIG. 2. Apparatus.

current up to 6000 A in the range between 0 and 140 v.

The rotor was mounted in four ball bearings so that an axial elongation of a few millimetres was possible. The housings of the ball bearings were electrically insulated from the heavy steel frame on which the apparatus rested.

The test section was rotated by a vee-belt drive from a direct current motor. The speed was controlled by regulating the motor field current and by changing the wheels of the belt drive. By means of this arrangement steady operation of the rotor was obtained for rotating speeds between 100 and 4000 rpm. Below 100 rpm fluctuations in the rotating speed occurred. and no measurements were therefore carried out below this value.

The water container was made from 5 mm thick stainless steel plates and was provided with two windows for visual observation of the flow around the rotating test section. In order to control the water temperature, two water coolers consisting of chromium-plated copper tubes with an outer diameter of 12 mm were placed in the container.

In order to determine the nondimensional numbers governing the heat-transfer rates for this type of flow, the following quantities had to be measured :

- 1. Outside wall temperature of the test section.
- 2. Surface heat flux of the test section.
- 3. Water bulk temperature.
- 4. Rotating speed of test section.

The outside wall temperature was obtained by measuring the temperature in the interior of the test section. This was achieved by means of a stationary thermocouple mounted inside a steel tube. 3 mm in diameter, which was inserted into a cavity of the rotor so that the thermocouple junction was located in the middle of the test section as shown in Fig. 1. The steel tube was supported by Teflon bearings mounted in the rotating part of the system. It should be emphasized that the thermocouple system is stationary. the test section rotating around it. In order to check the effects of axial conduction, the thermocouple was moved axially, during a few runs, and we found that isothermal conditions within ± 0.1 degC existed in the test section along almost its entire length. Axial conduction effects were only observed in approximately IO mm long stretches at the ends of the test section. From the thermocouple reading, which was identical with the inside wall temperature. t_{wi} , the outside wall temperature, t_{wo} , was evaluated by means of the equation

$$
t_{wo} = t_{wi} = \frac{q/A + d}{2\lambda} \left(\frac{d_i^2}{d^2 - d_i^2} \ln \frac{d}{d_i} - \frac{1}{2} \right). \tag{11}
$$

The water bulk temperature was measured by means of sixteen thermocouples placed inside stainless steel tubes located in the water container as shown in Fig. 3. The bulk temperature

FIG. 3. Location of thermocouples for measurements of water bulk temperature.

was taken as the average value of the thermocouple readings. Since all thermocouples except those two located just over the test section showed the same temperatures within $\pm 1 \text{degC}$. we found it necessary only to read the eight thermocouples which were nearest to the test section. For the measurement of the thermocouple voltages a precision Cambridge potentiometer was used. Ignoring the reading of the thermocouple just above the test specimen, the water bulk temperature was determined as the average of the remaining seven temperature readings. The thermocouple readings were also checked during a few runs by inserting a mercury thermometer in the tank of water. The two sets of readings agreed within ± 0.1 °C. The surface heat flux was determined from the equation

$$
q/A = \frac{R_1/R_2 \cdot EI}{\pi dL}
$$

where R_1 was the electric resistance of the test section and R_2 was the electric resistance of the rotor measured over the brushes. This ratio was O-978. The voltage over the brushes was measured with a Goerz precision voltmeter with a rated accuracy of $\frac{1}{4}$ per cent, and the current was obtained by measuring the voltage across a precision shunt calibrated to yield 60 mV at 3000 A. For this measurement a millivoltmeter with a rated accuracy of $\frac{1}{4}$ per cent was used.

The rotating speed was measured with a calibrated tachometer. For some of the runs the speed was also checked by counting the pulses which a small magnet mounted in the rotor induced in a stationary solenoid. The error of measured angular velocity was estimated at 1 per cent.

4. RESULTS AND DISCUSSIONS

157 runs were carried out. During these runs the cylinder rpm was varied from about 100 to 4000, corresponding to a rotating Reynolds number from about 1000 to 46 000. The water bulk temperature was varied between 15 and 65°C. It was not feasible to operate at much higher temperatures, since surface boiling should be avoided during this phase of the investigation, and the maximum surface temperature was therefore limited to about 100°C. Boiling effects have, however, also been studied and will be presented in a separate report [9].

All fluid properties were evaluated at the arithmetic mean of the surface and the bulk temperatures. The experimental results are summarized in Table 1 in terms of Nusselt, Reynolds, Grashof and Prandtl numbers.

As mentioned in an earlier section Anderson and Saunders **[l]** suggested that an analogy exists between the present problem and natural convection from a horizontal plate facing upwards. By employing the analogy they solved the problem for air. If the analogy is also applied to the general case of any fluid the following equation is obtained.

$$
Nu = 0.111 \; Re^{2/3} \cdot Pr^{1/3}.\tag{12}
$$

In Fig. 4, $Nu/Pr^{1/3}$ is plotted against the Reynolds number. The results show that in the range covered by the present investigation the effects of free convection are negligible since the Gra>hof number is not needed in order to correlate the

FIG. 4. Heat-transfer correlation for rotating cylinder.

data. The data are correlated by the equation

$$
Nu = 0.133 \; Re^{2/3} \cdot Pr^{1/3}.
$$
 (13)

and the deviation of the measurements from this equation is less than $+10$ per cent except for a few runs.

Equation (12) is also included in Fig. 4. The agreement between the theoretical solution and the measurements is rather good, the experimental results being about 20 per cent higher.

A comparison between equation (13) and the data of Seban and Johnson for oil and water is given in Fig. 5. The data for oil which covered Reynolds number from 0 to 15 000 and Prandtl numbers from 130 to 670 compare very well with the correlation for Reynolds numbers higher than 1000. Below this value the effects of free convection are appreciable so that some of the data are much higher than the correlation and also depend on the Grashof numbers.

Seban and Johnson's data for water compare excellently with the correlation for Reynolds numbers above \sim 3000. Below this value the rather high measured Nusselt numbers indicate free convection effects to be present. One should note that during the present measurements no free convection effects were observed at these Reynolds numbers. However, this observation is fully explained by considering that the Grashof numbers for Seban and Johnson's data are higher by a factor of approximately 20 compared to the present measurements,

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Table 1. Summary of experimental results

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Run No.	Re	Pr	$Gr \times 10^{-5}$	Nu	Run No.	Re	Pr	$Gr \times 10^{-5}$	Nu
113	2987	5.39	6.45	58.26	136	3405	2.92	12.37	42.45
114	3345	5.34	6.76	53.84	137	2359	2.81	16.75	33.44
115	995	3.36	29.62	19.01	138	1302	2.51	32.44	19.60
116	1669	4.08	12.95	32.86	139	4451	2.99	10.15	49.69
117	991	3.37	30.10	$19-48$	140	31729	2.93	$25 - 02$	194.41
118	2358	4.38	9.92	39.04	141	34788	2.96	22.74	211.34
119	3028	4.58	$7 - 75$	45.84	142	38284	2.95	21.37	224.10
120	2385	4.32	9.97	39.86	143	41237	2.98	19.66	239.37
121	1696	4.00	$13 - 75$	32.51	144	44243	$3 - 00$	18.59	250.88
122	1031	3.23	32.99	$19-20$	145	44486	2.99	18.71	253.51
123	3104	4.45	7.93	49.17	146	41774	2.94	19.89	242.62
124	1003	4.07	13.97	20.25	147	39039	2.89	22.10	226.66
125	1505	4.61	7.45	30.95	148	36207	2.84	24.32	209.16
126	2174	4.84	5.54	39.06	149	33481	2.77	$27 - 20$	195.06
127	2841	4.96	4.62	45.11	150	31925	2.91	23.16	196.14
128	2208	4.74	5.88	39.26	151	34870	2.95	21.24	$208 \cdot 10$
129	1552	4.45	8.85	29.11	152	37873	2.98	19.87	222.93
130	1018	4·00	$15-45$	20.88	153	44236	3.01	18.30	251.95
131	2869	4.90	4.96	47.96	154	41184	2.98	19.50	236.67
132	1301	2.52	32.87	19.91	155	46530	3.07	16.31	$269 - 42$
133	2371	2.79	17.26	32.69	156	44206	$2-01$	18.63	$250 - 44$
134	3420	2.91	$12 - 75$	$41 - 30$	157	41232	2.98	19.92	$236 - 78$
135	4438	2.99	9.96	50.39					

Table l-continued

FIG. 5. Comparison between the present results and the date of Seban and Johnson.

If the present results are extrapolated to a Prandtl number of 0.72 valid for air, equation (13) reduces to

$$
Nu = 0.119 \, Re^{2.3}.\tag{14}
$$

In Fig. 6 this equation is compared with the experimental equations mentioned earlier that were obtained for air.

On the average equation (14) yields about 15-25 per cent higher values than the other correlations for air.

The present data may also be used for testing the analogy solution by Kays and Bjorklund at different Prandtl numbers. Fig. 7 shows a comparison between the analogy solution and the present results for Prandtl numbers of 2 and

Fig. 6. Summary of experimental results for a horizontal cylinder rotating in air.

FIG. 7. Comparison between the present results and the analogy solution by Kays and Bjorklund.

results compare relatively well. compare well with each other.

In this paper consideration of heat transfer from a horizontal rotating cvlinder has been extended to the case of water. All measurements presented have been obtained in the region where the effects of natural convection are negligible and the heat-transfer rates depend' on the Reynolds and Prandtl numbers only.

On the basis of the experimental results a correlation in terms of Nusselt, Reynolds and Prandtl numbers has been established. This correlation compares very well with the available information in published works, and we conclude that experimental verification of the correlation has been found for Prandtl numbers between 0.72 and 670 and for Reynolds numbers up to 433 000 provided the effects of natural convection can be neglected.

Our results have been compared with the theoretical momentum and heat-transfer analogy solution of Kays and Bjorklund, and good agreement has been found to exist.

The analogy suggested by Anderson and Saunders between natural convection from a horizontal plate and the present problem has been used to analyse the problem. Analytical

5. The theoretical solution and the experimental and experimental results have been found to

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The author wish to record his appreciation of Mr. Henry 5. SUMMARY AND CONCLUSIONS Looft, who designed and built the apparatus and Mr.
his nonce consideration of heat transfer. Folke Wanche who participated in obtaining the experimental data

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Résumé—Cet article concerne les mesures des échanges thermiques à partir d'un cylindre horizontal en rotation dans l'eau. Les résultats expérimentaux sont bien représentés par l'equation $Nu = 0.133$ *Re^{2/3} Pr^{1/3} pour un domaine de nombre de Reynolds compris entre 1000 et 46.000 et des nombres* de Prandtl s'échelonnant de 2,2 à 6,4. Les données théoriques et expérimentales publiées pour l'air, l'eau et l'huile vérifient très bien cette équation.

Zusammenfassung-Die vorliegende Arbeit behandelt Messungen des Wärmeüberganges an einem in Wasser rotierenden waagrechten Zylinder. Die Versuchsergebnisse wurden durch die Gleichung $Nu = 0,133$ $Re^{2/3}$ $Pr^{1/3}$ korreliert für einen Bereich der Reynoldszahlen von 1000 bis 46000, und Prandtlzahlen von 2,2 bis 6,4. Diese Gleichung stimmt sehr gut uberein mit experimentellen und theoretischen Ergebnissen anderer Arbeiten für Luft, Wasser und Öl.

Die von Anderson und Saunders vorgeschlagene Analogie zwischen freier Konvektion an einer waagrechten Platte und der hier vorliegenden Strömungserscheinung diente zur Vorherbestimmung von Nusseltzahlen. Die AnalogielGsung und die vorliegenden Versuchsergebnisse lassen sich sehr gut miteinander vergleichen.

Аннотация-Статья посвящена теплообмену горизонтального цилиндра, вращающегося в воде. Экспериментальные данные обобщаются в виде уравнений $Nu = 0.133$ $Re^{2/3}$. $Pr^{1/3}$ при скоростях вращения, соответствующих числу Рейнольдса от 1000 до 46000 и числу Прандтля от 2,2 до 6,4. Это уравнение дает очень хорошее совпадение с экспериментальными и теоретическими данными, имеющимися в опубликованых работах, для воздуха, воды и нефти.

Аналогия, предложенаая Андерсоном и Саундерсом, между естественной конвекцией у горизонтальной пластины и рассматриваемым типом течения может быть применена для определения чисел Нуссельта. Решение с помощью аналогии и представленные в этой работе экспериментальные данные очень хорошо согласуются между собой.